R course - Exercice 2

First Name - Last Name

Master 2 Statistics and Econometrics

Problem

The objective is to compare two mechanisms of election in a fictitious electoral problem.

The context is as follows: we consider a country divided into 3 states, where each state contains the same number of voters n, where n is assumed to be odd. There are two candidates for the position of president: a candidate Py and a candidate R. We consider the following two mechanisms:

- Direct Mechanism (M1): The candidate with the most votes in the country wins the election.
- Indirect Mechanism (M2): Each state elects a representative Py or R (the one with the highest number of votes in the state). The party with the most representatives chooses its president.

Which criteria can be used for comparing the quality of an election?

We focus here on the probability that an elector is happy by the result. An elector is said to be happy if he/she has voted for the winner of the election.

Example :

We consider the following situation with 3 voters per state:

- in the 1st state, 3 people vote for Py and 0 votes for R
- in the 2nd state, 1 people votes for Py and 2 vote for R
- in the 3rd state, 1 people votes for Py and 2 vote for R

By considering M1, there were 5 votes for Py and 4 votes for R; in other words the election is won by Py. Here there are five voters happy, the ones who voted for Py.

By considering M1, 1st state was won by Py, 2nd state was won by R, 3rd state was won by R. In other words, R won the election with 2 representatives against 1 alone for the other side. The number of people happy is 4.

How to simulate the choice of an elector?

- **IC** case : a voter of the country chooses his candidate according to a Bernoulli of parameter $p = 1/2$.
- **IAC*** case: in each state k ($k = 1, ..., 3$), an elector chooses his candidate according to a Bernoulli of parameter p_k, where each p_k is simulated according to a Uniform law $[0, 1]$.

Remark: here, we are more interested in knowing the number of voters who voted Py or R within a state. Thus, instead of simulating in each state n Bernoulli, we can directly simulate the number of people who voted for Py using a Binomial of parameter (n, p) (function *rbinom()*) where p depends on the model above.

Objective

Create in **R** the function *simul_elec()*, having for input arguments:

- an integer **n** corresponding to the number of electors in a state,

- a string **case** which gives the simulation model of the choice of a voter,
- an integer **B** which gives the number of replications.

Inside the function, we will replicate **B** times the following simulation process:

- simulate according to **case**, the choice of voters in each state. The goal is to obtain a vector of size 3 (a value per state), containing the number of people who voted for one of the two candidates, choose Py for example, within each state. This can be done with the function *rbinom*().
- determine the winner of the election according to the mechanism **M1** or **M2**. Here, one always chooses Py as a reference, in other words one looks at whether or not Py won the election.
- keep in mind the following information for each mechanism: how many people are happy in **M1** and **M2**

The function will return for each mechanism, the average percentage on the B simulations of people satisfied by the election. In addition, it will check if possible, that the input arguments are adequate to the problem (**n** must be an odd and **case** a character equals to "IC" or "IAC_star")

We will take the following example to verify that the function works well:

```
res_IC <- simul_elec(n = 5, cas = "IC", B = 100000)
res_IAC_star <- simul_elec(n = 5, cas = "IAC_star", B = 100000)
```

Notation

You will have to return the exercice in *.pdf* or *.html* format, which would have been done with **R** Markdown if possible. It should contain the lines of code used to answer the questions, but you should also explain what you are doing.

Remark: try to use the least amount of command lines per question asked. The idea of this exercice is also to make you look for and find the most "elegant" and simple solutions to answer a given question. In particular, it will be possible to use *replicate()*, *switch()*, *rowMeans()*, *stopifnot()* functions.